UNIVERSITETET I AGDER

Emnekode : BE-409

Kandidatnr. : 1030

Dato : 29.09.2014

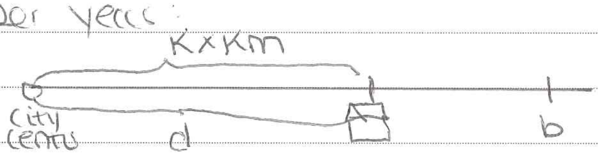
Ark nr. : 1 av ~~29~~ 25

Assignment 1

a) To show that the housingrent in a monocentric city can be expressed as $R(d) = (r\bar{q} + c) + k(b-d)$ I will use the Alonso-Muth-Mills model. This model helps us to find the rent at specific locations.

The assumptions in this model are:

* Every site/lot is unige. They are therefore not equal.
* The supply of space is priceinelastic
* The demand of space is priceelastic. It will therefore be the demandside that determines the price.
* We have a monocentric city where we only have one city center where all the businesses are located.
* The households commute a straight line at a cost k per km per year.



* The distance from the home to the city center is given by d.
* The city border is at a distance b from the city border
* All the houses are identical, and the housingrent is given by R(d)
* The households are identical, They use their income (y) on housingrent (R(d)), commuting costs (kd), and on other consumption (x^0).
* Every households uses q unit of land and have c in structure costs.

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Since all households are identical, we know that the income is used on:

$y = R(d) + Kd + x^0$

From this we can find an expression for the housing rent at a location d:

$R(d) = y - Kd - x^0$

At the city centre we have $d = 0$. The rent here will be:

$R(0) = y - x^0$

From this we can conclude that the housing rent is decreasing with increasing distance from the city centre, because of the component $-Kd$. People living at a distance d from the city centre must be compensated with lower housing rent because of the high commuting costs.

If we now consider a household that lives at the city border ($b = d$). We know that the alternative use of land at $d > b$ is r^a. So, at the city border the household will have to pay the sum of the agricultural rent (r^{ag}) and the construction/structure rent for the house (c). From this we can get:

$R(b) = y - Kb - x^0$

$(r^{ag} + c) = y - Kb - x^0$

From the assumptions in this model we have that all of the households are equal. They will therefore have equal consumption of other goods:

$x^o = y - Kb - (r^a q + c)$

The housing rent at a distance d from the city center can therefore be expressed:

$R(d) = y - Kd - x^o$
$R(d) = y - Kd - y + Kb + (r^a q + c)$
$R(d) = Kb - Kd + (r^a q + c)$
$R(d) = (r^a q + c) + K(b-d)$

So, the housing rent therefore consists of three components:
① $r^a q$ → The agricultural rent
② c → The structure rent
③ $K(b-d)$ → The location rent (saved commuting costs.)

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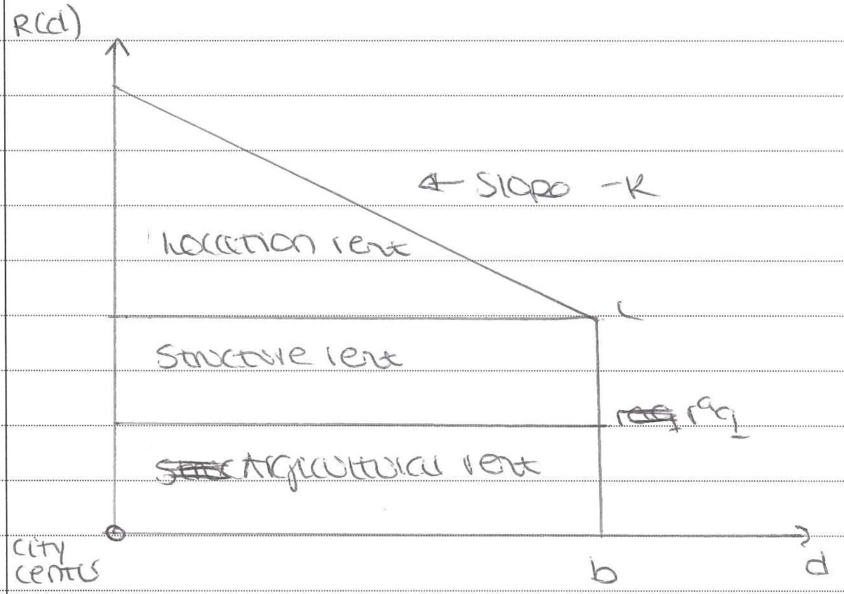
b) The answer from a) can be shown in a figure with distance from the city centre at the x-axis and the housing rent at the y-axis:



Households living at a distance d from the city centre most be compensated for the commuting costs
⇒ They therefore pay a lower housing rent

As we know and can see from the figure, the argicultural rent and the structure rent are constant at all locations. The only variation in rent at different locations will therefore be the location rent. The slope of this rent is what we call the rent gradient

$$\frac{\partial R(d)}{\partial d} = -K$$

That is, the higher the commuting costs (K) is, the higher/steeper will the rent gradient be. The households living at a distance d from the city centre will pay a relativly lower rent than the households living in the city centre.

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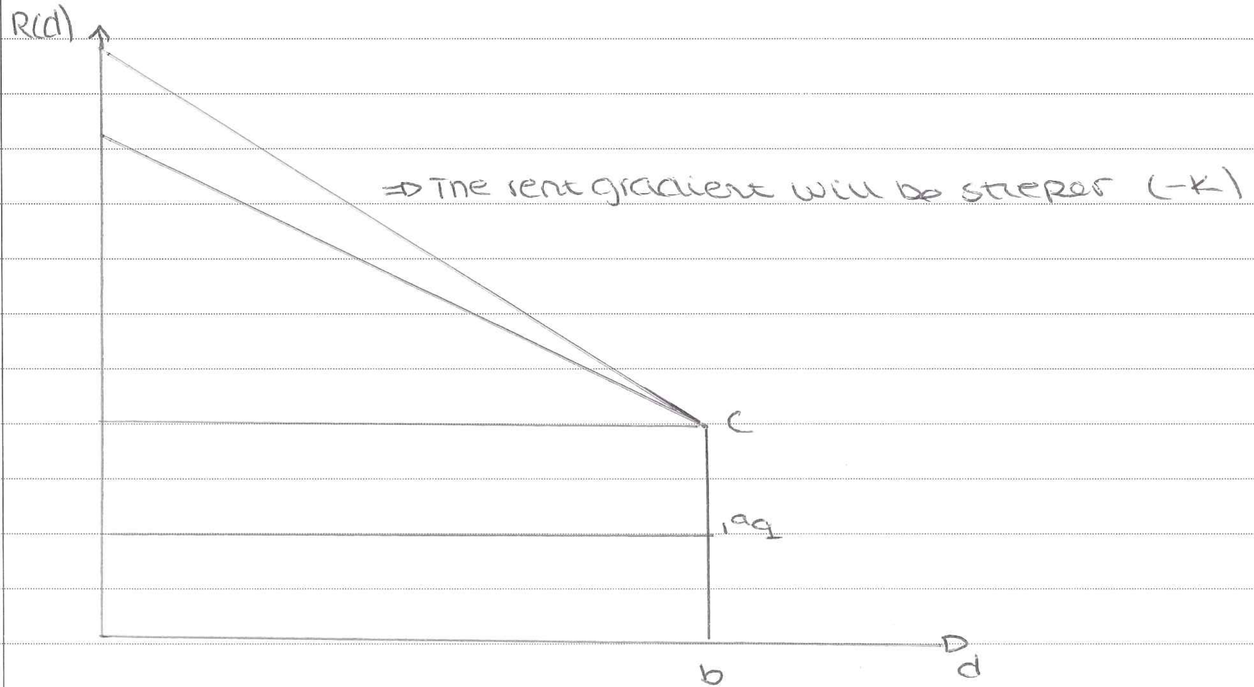
The housing rent R(d) at the city centr will always be the highest due to the negative slope of the rent gradient => They have no cost of commuting. If we consider a case where the commuting costs are increasing, the rent gradient will be steeper!



And the housing rent will increase more in the city center than the other locations within the city.

c) The land rent will only consist of two components:
① The agricultural rent
② The location rent

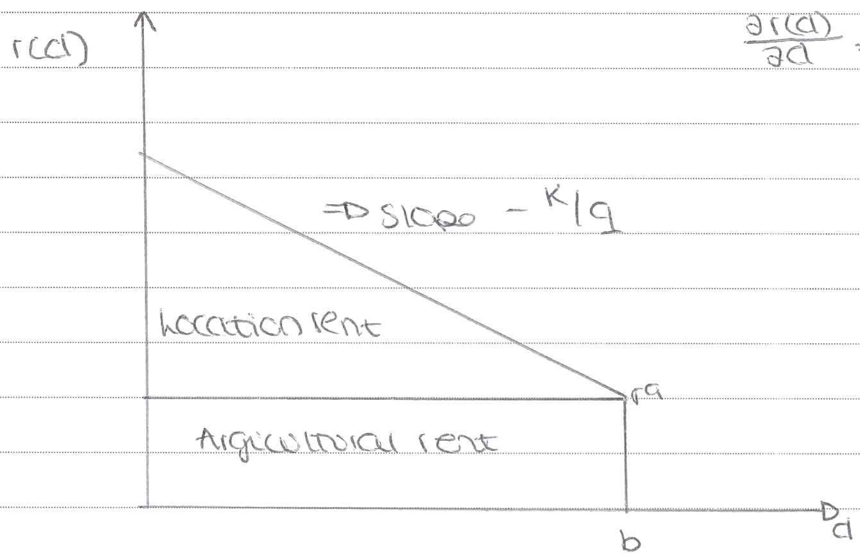
The housing rent gives us the rent per household, but the land rent will be given per acre ⇒ so we have to divide by q and subtract the structure rent to find the land rent r(d):

$$r(d) = (R(d) - c)/q$$
$$r(d) = [r^a q + c + k(b-d) - c]/q$$
$$= [r^a q + k(b-d)]/q$$
$$= \frac{r^a q}{q} + \frac{k(b-d)}{q}$$

$$\Rightarrow r(d) = r^a + \frac{k(b-d)}{q}$$

So, the land rent will consist of the alternative use of land, and the saved commuting cost at a place d per acre (a m^2).

$$\frac{\partial r(d)}{\partial d} = -\frac{kq}{q^2} \Rightarrow -k/q$$



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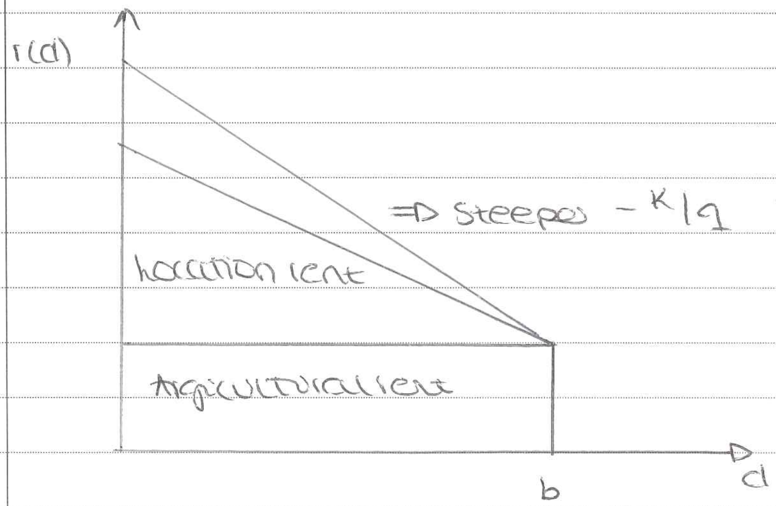
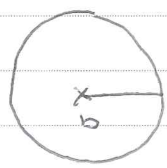
d) The rent on land will increase when the rent gradient $-K/q$ increase. This will be when the commuting costs (K) increase cet.par, or when the density increases (i.e lower q) cet.par, or when we have both cases occuring at the same time. The land rent gradient will be steeper:



We can see that this will increase the land rent at every location within the city border (at $d<b$). The increase in the land rent will be bigger the closer to the city center we are.

An expansion / increase in the city border will also increase the land rent. We assume that we have a circular city where $V=1$ ⇒ full circular city
$V=1/2$ ⇒ half circular city



V will therefore stand for the supply of land, b will be the city border, n will be the number of household and q will be the amount of land that every household needs. The supply of land have to

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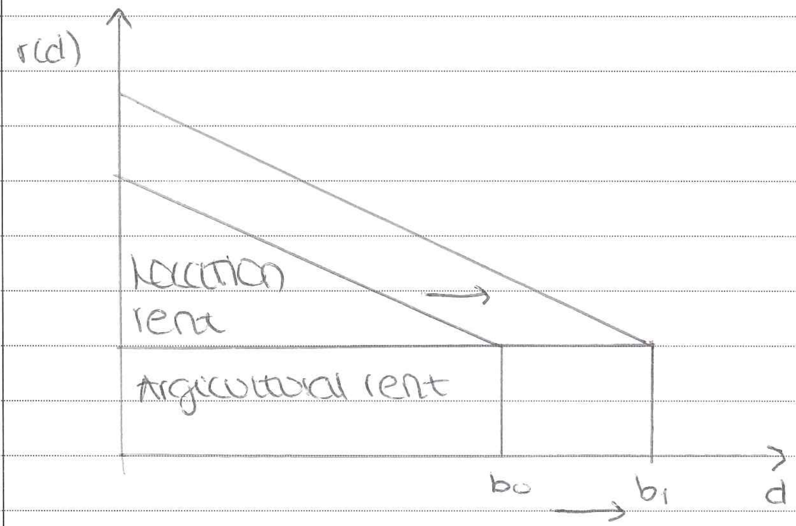
equal the demand for land:

$b^2 \cdot V \cdot \pi = nq \quad \Rightarrow b = \sqrt{\frac{n \cdot q}{V \cdot \pi}}$

The city border will therefore expand when:
→ The number of households increases
→ The amount of land that every household uses increases
→ If the supply of land decreases



When the city border grows from b_0 to b_1 we will get an increase in the land rent of every location within the new city border.

Also, if the alternative use of land increases the agricultural rent will increase for all locations and the land rent will increase.

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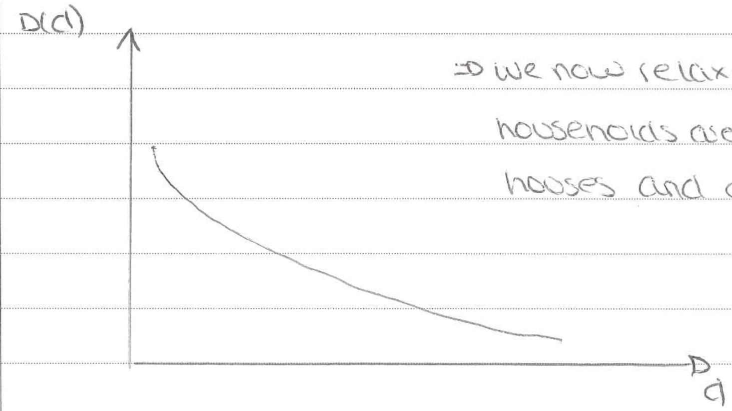
e) Now the density gradient are no longer flat. The density will now be higher where the price of land is high. This is because the construction now substitute away from the expensive good (land) and use more of the "cheap" good wich is construction.

The density at a distance d will now be decided from a two-variable regression model.

$D(d) = D_0 \cdot e^{-\alpha d}$

This tells us that the density gradient is no longer flat:



⇒ We now relax the assumptions that the households are identical w.t. identical houses and equal use of space.

Now to the model: we assume that the landowner wants to maximize the profit from land after the construction has been made. The land is for residential use only.

We also assume that the w.t.p is decreasing with increasing density. The profit per unit will therefore be lower, but the number of units will be higher.

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The wtp is given by: $P = \alpha - \beta F$
where α is the general wtp when the density is zero. The wtp is decreasing with dencity, so the coefficient β shows the reduction in the wtp when the density ~~...~~ increases with one unit.

The costs is given by: $C = \mu + \tau F$
where μ is the cost of constructing when the density is zero. The construction costs will increase with density. The coefficient τ show the increase in the construction costs when the density increases.

The profit for the housing floor area will be $\Pi_{BOA} = P - C$
The landowner wants to maximize the profit from the land:

$$\Pi_{BOA} = (P - C) \cdot F$$
$$= (\alpha - \beta F - \mu - \tau F) \cdot F$$
$$= \alpha F - \beta F^2 - \mu F - \tau F^2$$
$$= (\alpha - \mu) F - (\beta + \tau) F^2$$ $\Rightarrow$ This will be a parabel

The optimal profit will be at the maximum point of this parabel:

$$\frac{\partial \Pi}{\partial F} = (\alpha - \mu) - 2F(\beta + \tau) = 0$$
$$(\alpha - \mu) = 2F(\beta + \tau)$$

$$F^{opt} = \frac{(\alpha - \mu)}{2(\beta + \tau)} \qquad profit^{opt} = \frac{(\alpha - \mu)^2}{4(\beta + \tau)}$$

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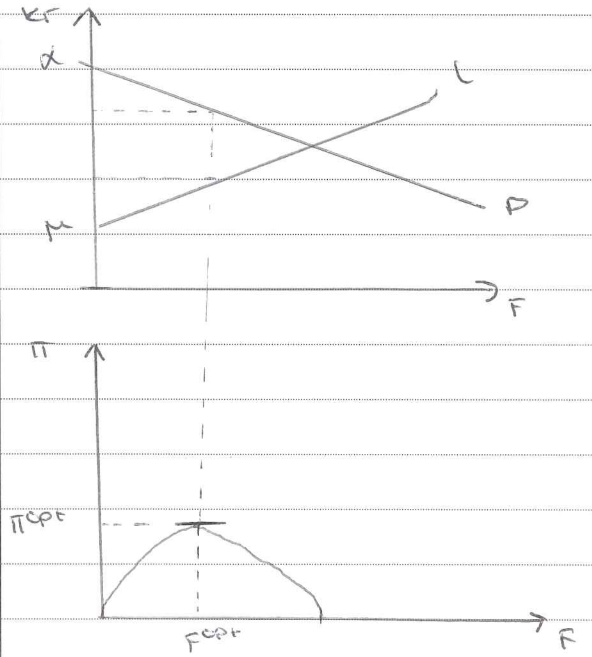

The optimal density will be F^{opt} when the landowners profit are maximized!

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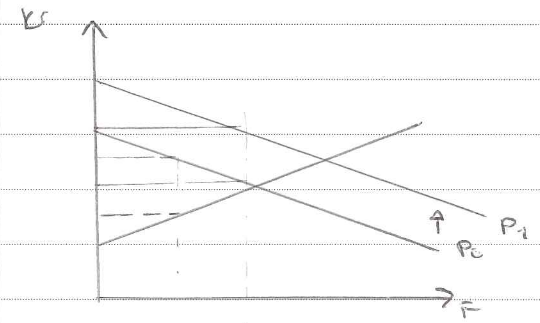
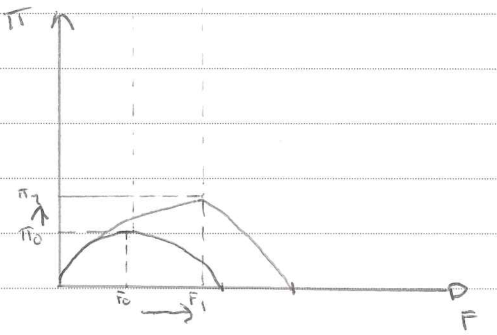
f) if the attractivness of the landowners plot increases the demand for this plot will go up and the w.t.p for this plot will increase

$P = \alpha - \beta F$

This tells us that α will increase, and we will get a shift in the price-line:



From the figure we can see that the price-line get a positive shift. This increase in the willingness to pay will increase the landowners profit and we will get ~~an~~ an increase in the optimal density.



The profit increases from π_0 to π_1 and the optimal density increases from F_0 to F_1.

$$F^{opt} = \frac{\overset{\rightarrow \text{increase}}{(\alpha - \mu)}}{2(\beta + \tau)} \Rightarrow \text{so } F^{opt} \text{ increase}$$

$$\pi^{opt} = \frac{\overset{\rightarrow \text{increase}}{(\alpha - \mu)}}{4(\beta + \tau)} \Rightarrow \text{so } \pi^{opt} \text{ increase.}$$

g) Now we have external effects across properties.
$f \rightarrow$ The neighbourhood F
$\gamma \rightarrow$ The marginal loss in value due to increased f

In an unregulated market the profit of the landowner will now be:

$$\Pi = (P - C - \gamma f) \cdot F$$
$$= (\alpha - \beta F - C - \gamma f) F$$
$$= \alpha F - \beta F^2 - CF - \gamma f F$$

The landowner maximizes the profit for land, so the optimal density in an uncoordinated market will be:

$$\frac{\partial \Pi}{\partial F} = (\alpha - C - \gamma f) - 2F\beta = 0$$

$$2F\beta = (\alpha - C - \gamma f)$$

$$F = \frac{(\alpha - C - \gamma f)}{2\beta}$$

we can see that the optimal density and thus the optimal profit will be reduced due to $-\gamma F$. In the long run we will have $f = F$, so the optimal density in the uncoordinated market in the long run will be:

$$F = \frac{(\alpha - C - \gamma F)}{2\beta}$$
$$F 2\beta + \gamma F = \alpha - C$$
$$F(2\beta + \gamma) = \alpha - C$$
$$F^m = \frac{\alpha - C}{2\beta + \gamma}$$

$\Rightarrow$ The profit will be $\Pi^m = \beta \left(\frac{\alpha - C}{2\beta + \gamma}\right)^2$

In the long run we can see that the external effects in the neighbourhood f will be equal to F. This will give us a lower optimal density due to the external effects such as noise, smell etc. The optimal profit will also be reduced.

h) Such unregulated development of land, will as we saw under f reduce the landowners profit and reduce the density. To look at the consequences of a such uncoordinated market, we could look at how the density will be at a coordinated market (when $f=F$)

$$\pi = (P - C - \gamma F)\cdot F$$
$$= (\alpha - \beta F - C - \gamma F)\cdot F$$
$$= \alpha F - \beta F^2 - CF - \gamma F^2$$
$$= (\alpha - C)\cdot F - (\beta + \gamma)F^2$$

$$\frac{\partial \pi}{\partial F} = (\alpha - C) - (\beta + \gamma)\cdot 2F = 0$$
$$2F(\beta + \gamma) = (\alpha - C)$$

$$F^* = \frac{(\alpha - C)}{2(\beta + \gamma)} \qquad \pi^* = \frac{(\alpha - C)^2}{4(\beta + \gamma)}$$

We can from this see and conclude that the density will be lower in a coordinated market, than a uncoordinated market (because of $2\gamma > \gamma$), but the profit will be higher in the coordinated market than in the uncoordinated market. So, a unregulated market will not be the best way in the long run, because then we will get a higher density and a lower profit

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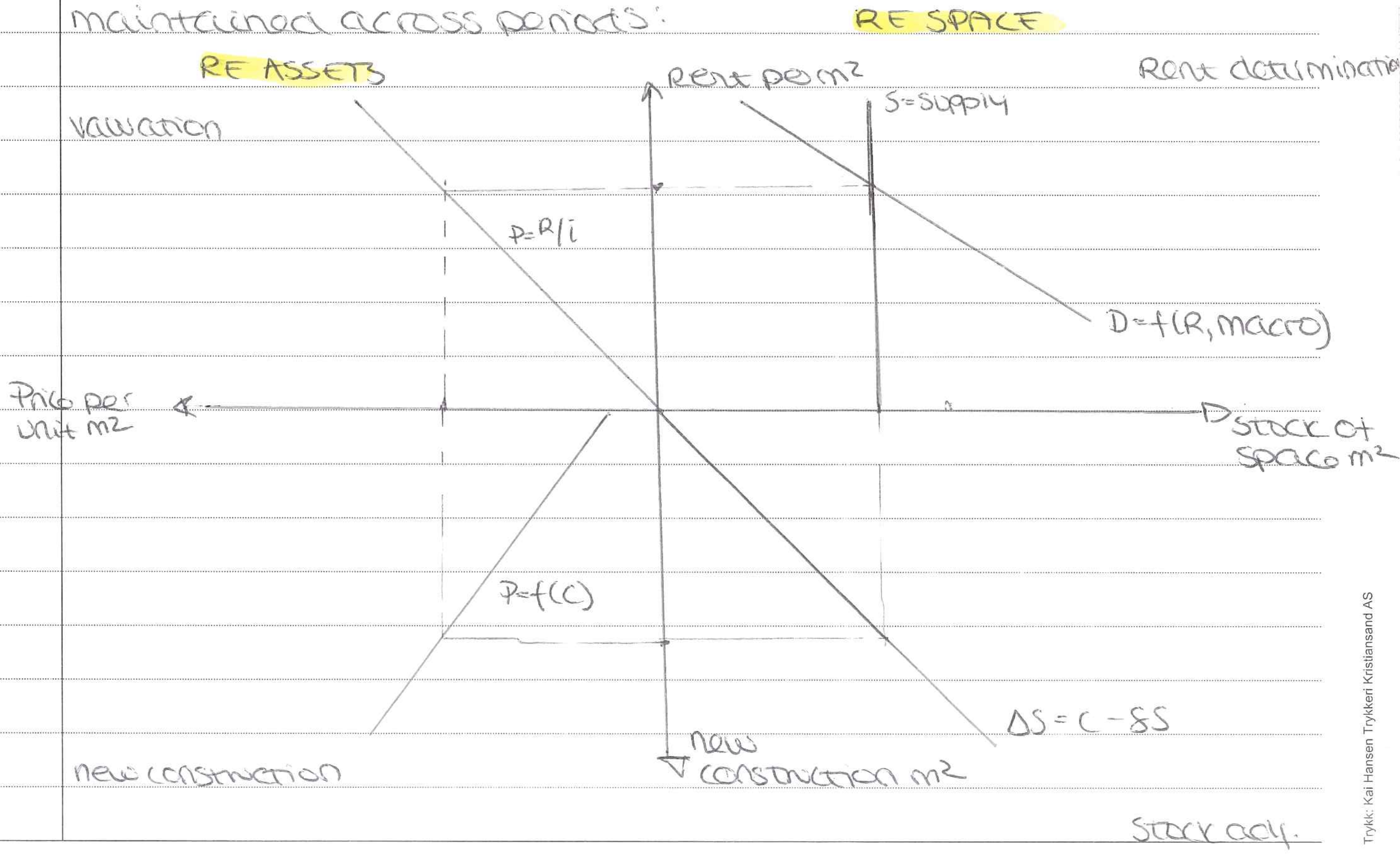
ASSIGNMENT 2

a) The DiPasquale and Wheaton model show us the relationship between the property and the capital/asset market. We will assume that we have two markets:
① The market for real estate use (assume that we have tenants)
② The market for real estate assets.

This model will show the connection between the property market (Real estate use), the capital market (Real estate assets) and new construction.

First we will assume that the demand will be the same wheater you are a owner or a tenant, a household or a firm. The supply will be inelastic in the short run and consist of the stock of space. And equilibrium can be maintained across periods:



On the rightside of this figure we have the market for real estate space / property market. On the left side of this figure we have the Real estate assets / ~~property~~ capital market.

In the upper right corner we have the rent determination, here the supply of space is given and the demand is determined by the rent (R) and on other economic factors: D = f(R, macro). So, for this market to be in equilibrium the stock of space have to equal the demand.

In the upper left corner we have the valuation, given the rent determined in the property market we can find the price per unit of m^2 by using the ~~capitita~~ capitalization rate: $P = R/i$ where i is the yield that the investors demand. The price will just be the future flow of income (Rent) from the property market.

In the lower left corner we have the market for new constructions. The price in the capital market will together with the supply of new construction decide the new constructions. For this market to be in equilibrium the price per unit have to equal the replacement cost of construction: $P = f(C)$. The higher the price is, the higher will the level of new construction be.

In the lower right corner we have the stock adjustments. The change in the stock of space, ΔS, will be given by new constructions (C) subtracted the depreciation of already existing buildings δS, which is the reduction of stock:

$\Delta S = C - \delta S$

When the new stock of space is equal to the old stock of space the model is in complete equilibrium. For this to be true we have to have $\Delta S = 0$. This is when the new construction is equal to the depreciation of existing stock of space $C = \delta S$

So, the property market will give us a price in the capital market, because the capital market is just really future income of rent in the capital market. This price will again decide the level of new construction, which again decide the stock adjustment => the supply of stock of space.

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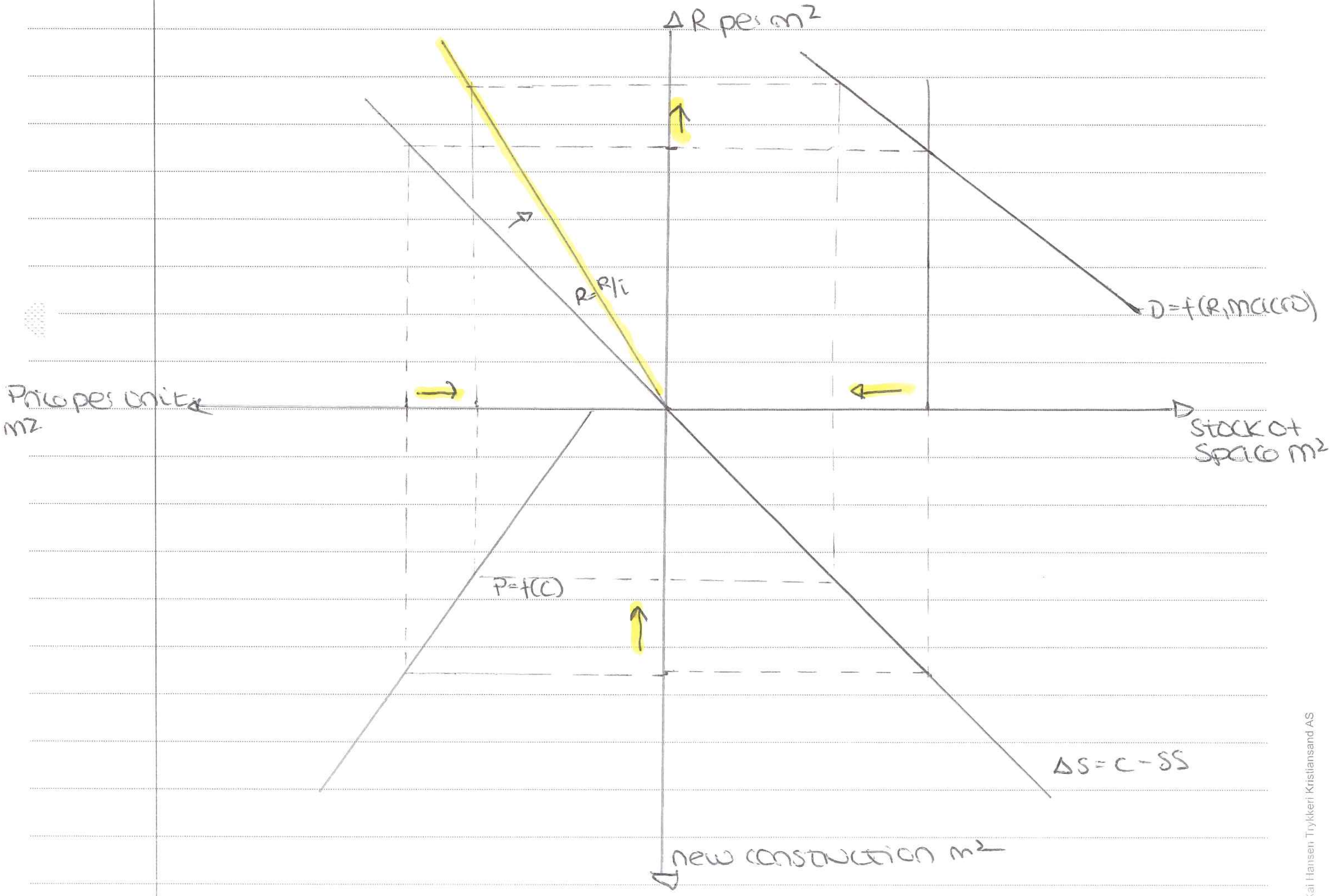
b) The long run equilibrium effect of a general increase in the capitalization rate will lead to a clockwise rotation in the rent-to-price ratio line in the capital market. The capitalization rate is ~~eg~~ eksogenously given. In general a increase in the capitalization rate can be because of :

* increased interest rates
* Increased uncertainty about future rent income
* Increased property tax etc.

We will get a clockwise rotation in the capitalization rate : $P = R/i$



The increase in the capitalization rate will lead to a decrease in the price per unit of m^2, which again will lead to a decrease in the ~~E~~ level of new construction. This reduction in construction will again lead to that the stock of space decreases because of the lower construction. The new construction will not be large enough to cover the depreciation of already existing stock. This leads to that the supply (stock of space) decreases and the rent will increase because of the lower supply!

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assignment 3

a) In the Alonso-Muth-Mills model we assumed that we had a monocentric city with one city center and no subcenters. We here assumed that all the businesses was located in the city center. For this to be true the businesses have to utilise central land more than households do.

We assume:

* monocentric city with one city center where all the firms are located.
* The distance from the city center is d
* The city border is at a distance b from the city center
* All the goods are imported and exported from the city center.
* We have perfect competition, so the prices are given
* We have transportation costs of s per km within the city
* The firms are identical, they produce Q units and uses the same production technology
* The firm ~~E~~ faces equal cost C and use the same space f.
* The land rent is $r_c(d)$
* The land is rented out to the highest paying firm

The profit of each firm will then be given by:

$\pi = Q(P - A - sd) - C - r_c(d) \cdot f$

Where:

Q - quantum produced and sold

A - variable cost

sd - transp. cost

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When we have perfect competition, the profit of each firm will be zero:

$$r_c(d) \cdot f = Q(P-A-sd) - C$$

$$r_c(d) = \frac{Q(P-A-sd) - C}{f}$$

The land rent for each firm will then be decreasing, with increasing transportation costs $(-sd)$. The rent gradient for the firms will be:

$$\frac{\partial r_c(d)}{\partial d} = -\frac{Qs\cdot f}{f^2} = -\frac{sQ}{f}$$

If we now allow for different firms (i) with different transportation cost (S), different produced and sold quantum (Q) and different use of space (f), then the rent gradient will look like this:

$$\Rightarrow -\frac{S_i\, Q_i}{f_i}$$

So firms with a high rent gradient will locate at the city center, and firms with a low rent gradient will locate at the city outskirts:

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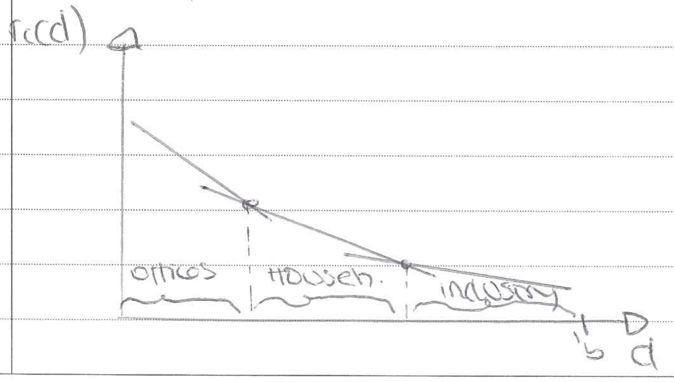
So, a firm will choose to locate at the city outskirts when they have low transportation costs of goods (s), and they have a low Q/f => produce little per m². This will be when the firms has lower transportation costs than the households.

b) I have explained this model under a). So why offices are located in the city centre and industry is located at the city outskirts will be because of the rent gradient for offices is much higher than the rent gradient for industry.

The offices has high transportation costs (s) and produce much per square feet (high Q/f). This slope will be higher than the households rent gradient when their transport costs is higher than the household. This is when f.ex the households valuation of time is low.

Because of increased transportation technology the industry firms often have lower cost due to transportation of goods. Industry firms often also require a lot of space. Both this components will decrease the rent gradient and they will choose to locate at the city outskirts.



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we now allow for the formation of subcenters

c) The agglomeration ~~costs~~ benefits from clustering can be that it will be easier to communicate with different divisions in the same firm. This benefit is a bit reduced today because of all of the good communication technology that we have today

The other benefit from firms clustering is that it becomes easier to communicate with the other firms in that area. To see what the other firms are doing will be a benefit.

d)

*we assume that the wage will grow at a constant rate as the subcenter increases. This is because the workers has to be compensated for higher commuting costs as the subcenter increases

*we assume that the agglomeration costs will be less as the subcenter increases. This will however not be a linear relationship. The agglomeration costs will at the end go against the x-axis

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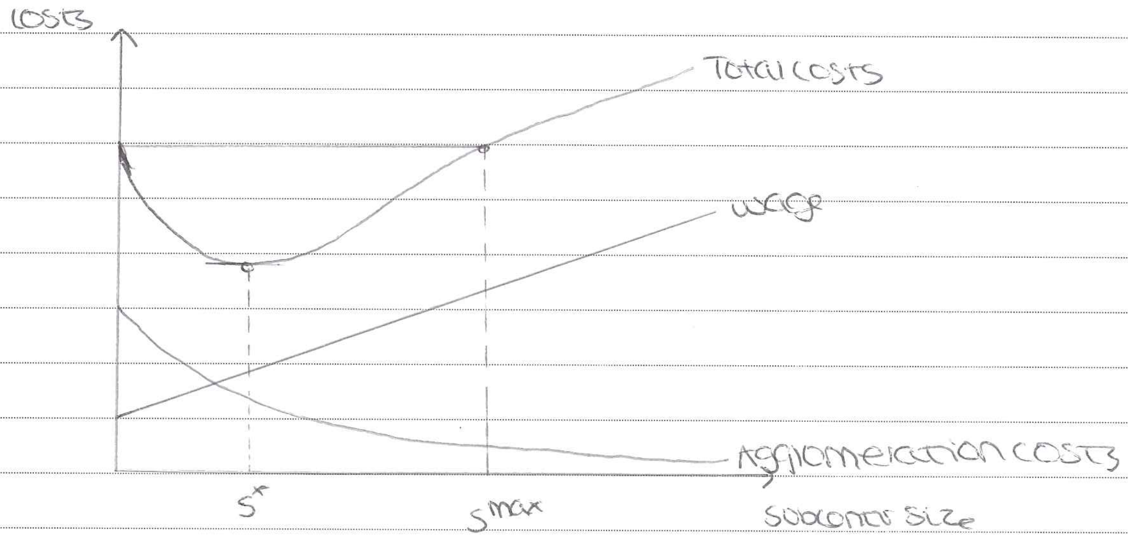

This is not a very good drawing, but we can see that the optimal size of the subcenter will be when the total costs are at its lowest.

e)

when a subsenter is reality it will grow until it reaches its optimal size s*, it will continue to grow ~~unit~~ until the total costs is equal to the "total starting costs". Now the size of the subcenter is s max.

A new subcenter will be formed and it will grow the same way. when the second subcenter reaches its optimal size, firms at subcenter "1" will not move to subcenter "2" because of the sunkcost. Subcenter 2 will continue to grow unit it reaches the size s max.

A 3rd subcenter will then be formed!

Subcenters are formed when the firms rent gradients not are so high, so they will be willing to locate near the city edge.

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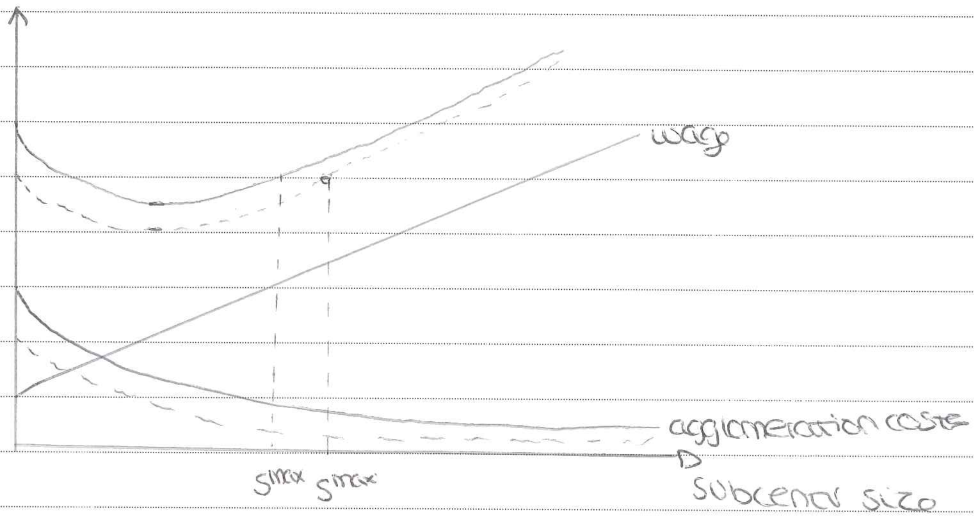
In d) If the agglomeration benefits from clustering in a subcenter gets higher, the agglomeration costs will get lower



The subcenter will therefore get the same optimal size S*, but since the total costs of the subcenter is decreasing the maximum size of the subsentr will increase. The center will therefore be bigger the smaller the agglomeration costs are

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